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Vector beams as the superposition of cylindrical partial waves in bianisotropic media

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Abstract

The exact solutions for arbitrary electromagnetic beams in bianisotropic media are constructed. The solutions are expressed using tensor Fourier transform whose physical meaning is the superposition of partial waves. We use cylindrical partial waves (vector Bessel beams) and derive exact and paraxial solutions for cylindrically symmetric beams in isotropic, bi-isotropic and bianisotropic media. The comparison of the spatial evolution of vector Bessel–Gauss beams in different media is made.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Maxwell's equations can be written in such coordinate frames as Cartesian, cylindrical and spherical coordinates. The general solution of Maxwell's equations is expressed by means of a superposition of partial waves. In our paper cylindrical eigenwaves are used. The choice of such waves is caused by wide applications of cylindrically symmetric solutions in electrodynamics. For instance, cylindrical waves describe Bessel beams [1, 2] and electromagnetic radiation in circular fibres [3, 4].

There is the extensive class of bianisotropic media whose cylindrical eigenwaves are described by the Bessel functions. Usually, such eigenwaves are called vector Bessel beams [5–7]. Bessel beams can differ both in their order and transverse (radial) wavenumber. A beam with some field distribution in its cross-section can be decomposed into cylindrical partial waves (eigenwaves with different wavevectors), each of which does not change the transverse field distribution during propagation, i.e. it is diffraction free. However, the superposition of eigenwaves with differing transverse wavenumbers diffracts and can describe the spatial evolution of Bessel–Gauss [8–11], Laguerre–Gauss [12, 13] and Hermite–Gauss [14] beams.

So, the solution for an arbitrary beam can be written by means of the Fourier transform. In the case of cylindrically symmetric solutions the transform is often called the Fourier–Bessel transform. In papers [15, 16], the Fourier transform was utilized to study paraxial propagation of cylindrically symmetric beams in uniaxial crystals.

The goal of the paper is to present the method for investigation of cylindrically symmetric beams in a wide class of bianisotropic media. In section 2 of the present investigation the method for calculating electric and magnetic fields of cylindrical eigenwaves in complex media is given. Expressing the transverse field components by means of tensors we find the main properties of vector Bessel beam solutions. Section 3 is devoted to the determination of exact and paraxial solutions for electromagnetic beams with arbitrary field distribution in bianisotropic media. In section 4 the beams in isotropic, bi-isotropic and bianisotropic media are studied. The analysis of exact and paraxial cylindrically symmetric beams is carried out. In section 5 the polarizations and intensities of Bessel–Gauss beams in isotropic, bi-isotropic and bianisotropic media are compared.

2. Cylindrical eigenwaves in bianisotropic media: vector Bessel beams

Bianisotropic media are described by general constitutive equations, which include not only dielectric permittivity ε and magnetic permeability μ tensors, but gyration tensors α and κ as well:

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} + \alpha \boldsymbol{H}, \qquad \boldsymbol{B} = \kappa \boldsymbol{E} + \mu \boldsymbol{H}, \tag{1}$$

where $\boldsymbol{H}, \boldsymbol{E}, \boldsymbol{B}$ and $\boldsymbol{D}$ are strengths and inductions of magnetic and electric fields. In [17] constitutive equations for bianisotropic media were used for describing optical activity (gyrotropy) of materials. General constitutive equations (1) allow us to describe some special cases, such as bi-isotropic media, gyrotropic Faraday materials and anisotropic media. Bianisotropic media can be created on the basis of composite materials [18]. It should be noted that strongly investigated negative-refractive-index media have the same origin [19]. However, we can regard them as isotropic media, when the wavelength is much greater than a typical period of a material.

Here, we consider only tensors of the form

$$\xi = \xi_1 I_z + \xi_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_z + \mathrm{i}\chi_\xi \boldsymbol{e}_z^\times, \tag{2}$$

where (r, φ, z) are cylindrical coordinates, ξ corresponds to one of the tensors $\varepsilon, \mu, \alpha, \kappa$, $\boldsymbol{e}_1 = \boldsymbol{e}_r(\varphi), \boldsymbol{e}_2 = \boldsymbol{e}_\varphi(\varphi), \boldsymbol{e}_3 = \boldsymbol{e}_z$ are the basis vectors in cylindrical coordinates, $\boldsymbol{e}_i \otimes \boldsymbol{e}_j$ is the dyad, $\boldsymbol{e}_z^\times$ is the tensor dual to the vector $\boldsymbol{e}_z$ [17, 20, 21], $I_z = 1 - \boldsymbol{e}_z \otimes \boldsymbol{e}_z$ is the projection operator onto the plane perpendicular to vector $\boldsymbol{e}_z$. There are some reasons to choose tensors $\varepsilon, \mu, \alpha, \kappa$ in the form (2). First, equation (2) is enough general for description of a number of bianisotropic, gyrotropic and anisotropic media. Second, such tensors $\varepsilon, \mu, \alpha, \kappa$ are simplest for technological applications, because they have single optic axis $\boldsymbol{e}_z$. Third, tensors $\varepsilon, \mu, \alpha, \kappa$ in the form (2) allow us to make calculations easily, because cylindrical eigenwaves propagating in such a medium retain cylindrical symmetry during their propagation and can be expressed by the Bessel functions. Cylindrical eigenwaves are the stationary fields with the following variable separation:

$$\begin{pmatrix} \boldsymbol{H}(\boldsymbol{r}, t) \\ \boldsymbol{E}(\boldsymbol{r}, t) \end{pmatrix} = \mathrm{e}^{\mathrm{i}\beta z + \mathrm{i}\nu\varphi - \mathrm{i}\omega t} \begin{pmatrix} \boldsymbol{H}(r) \\ \boldsymbol{E}(r) \end{pmatrix}, \tag{3}$$

where β is the longitudinal wavenumber, ω is the wave frequency and ν is the azimuthal number taking integer values. As a result, from the Maxwell equations it follows the system of ordinary differential equations of the first order [20] for tangential field components

$$\frac{\mathrm{d}\boldsymbol{W}(r)}{\mathrm{d}r} = \mathrm{i}kM(r)\boldsymbol{W}(r), \tag{4}$$

where

$$\begin{gathered}
M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \qquad \boldsymbol{W} = \begin{pmatrix} \boldsymbol{H}_{\mathrm{t}} \\ \boldsymbol{E}_{\mathrm{t}} \end{pmatrix} \\
A = \frac{\mathrm{i}}{kr}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi + \boldsymbol{e}_r^\times \alpha I_r + \boldsymbol{e}_r^\times \varepsilon \boldsymbol{e}_r \otimes \boldsymbol{v}_3 + \boldsymbol{e}_r^\times (\boldsymbol{u} + \alpha \boldsymbol{e}_r) \otimes \boldsymbol{v}_1 \\
B = \boldsymbol{e}_r^\times \varepsilon I_r + \boldsymbol{e}_r^\times \varepsilon \boldsymbol{e}_r \otimes \boldsymbol{v}_4 + \boldsymbol{e}_r^\times (\boldsymbol{u} + \alpha \boldsymbol{e}_r) \otimes \boldsymbol{v}_2 \\
C = -\boldsymbol{e}_r^\times \mu I_r - \boldsymbol{e}_r^\times \mu \boldsymbol{e}_r \otimes \boldsymbol{v}_1 + \boldsymbol{e}_r^\times (\boldsymbol{u} - \kappa \boldsymbol{e}_r) \otimes \boldsymbol{v}_3 \\
D = \frac{\mathrm{i}}{kr}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi - \boldsymbol{e}_r^\times \kappa I_r - \boldsymbol{e}_r^\times \mu \boldsymbol{e}_r \otimes \boldsymbol{v}_2 + \boldsymbol{e}_r^\times (\boldsymbol{u} - \kappa \boldsymbol{e}_r) \otimes \boldsymbol{v}_4 \\
\boldsymbol{v}_1 = \delta_r(\kappa_{rr}\boldsymbol{e}_r \alpha I_r - \varepsilon_{rr}\boldsymbol{e}_r \mu I_r - \kappa_{rr}\boldsymbol{u}) \qquad \boldsymbol{v}_2 = \delta_r(\kappa_{rr}\boldsymbol{e}_r \varepsilon I_r - \varepsilon_{rr}\boldsymbol{e}_r \kappa I_r - \varepsilon_{rr}\boldsymbol{u}) \\
\boldsymbol{v}_3 = \delta_r(\alpha_{rr}\boldsymbol{e}_r \mu I_r - \mu_{rr}\boldsymbol{e}_r \alpha I_r + \mu_{rr}\boldsymbol{u}) \qquad \boldsymbol{v}_4 = \delta_r(\alpha_{rr}\boldsymbol{e}_r \kappa I_r - \mu_{rr}\boldsymbol{e}_r \varepsilon I_r + \alpha_{rr}\boldsymbol{u}) \\
\boldsymbol{u} = (\beta/k)\boldsymbol{e}_\varphi - \nu/(kr)\boldsymbol{e}_z \qquad \delta_r = (\varepsilon_{rr}\mu_{rr} - \alpha_{rr}\kappa_{rr})^{-1} \\
\varepsilon_{rr} = \boldsymbol{e}_r \varepsilon \boldsymbol{e}_r \qquad \mu_{rr} = \boldsymbol{e}_r \mu \boldsymbol{e}_r \qquad \alpha_{rr} = \boldsymbol{e}_r \alpha \boldsymbol{e}_r \qquad \kappa_{rr} = \boldsymbol{e}_r \kappa \boldsymbol{e}_r.
\end{gathered} \tag{5}$$

Tangential components of strength vectors are situated in the plane (φ, z) and equal to $\boldsymbol{E}_{\mathrm{t}} = I_r \boldsymbol{E}$ and $\boldsymbol{H}_{\mathrm{t}} = I_r \boldsymbol{H}$, where $I_r = 1 - \boldsymbol{e}_r \otimes \boldsymbol{e}_r$ is the projection operator onto the plane orthogonal to the unit vector $\boldsymbol{e}_r$, $k = \omega/c$ is the vacuum wavenumber. We should note that the system of differential equations of the first order for planar stratified media was derived earlier (see, for example, papers [22, 23]).

As derived in [20], the longitudinal field components of the wave propagating in a bianisotropic medium (2) yield to the differential equation

$$\frac{\mathrm{d}^2}{\mathrm{d}r^2}\begin{pmatrix} H_z \\ E_z \end{pmatrix} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\begin{pmatrix} H_z \\ E_z \end{pmatrix} + \left(Q - \frac{\nu^2}{r^2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)\begin{pmatrix} H_z \\ E_z \end{pmatrix} = 0, \tag{6}$$

where matrix Q is determined by the medium parameters and wavenumbers k, β. It is obvious that the solutions of equation (6) are the Bessel functions of the first J_ν and second Y_ν kind of the order ν. Using the spectral decomposition for matrix Q one obtains

$$\begin{pmatrix} H_z \\ E_z \end{pmatrix} = (c_1 J_\nu(q_1 r) + c_3 Y_\nu(q_1 r))\vec{w}_1 + (c_2 J_\nu(q_2 r) + c_4 Y_\nu(q_2 r))\vec{w}_2, \tag{7}$$

where q_1^2, q_2^2 are the eigenvalues of Q and $\vec{w}_1, \vec{w}_2$ are its eigenvectors. If we unite the constants $c_i, i = 1, 2, 3, 4$, into two vectors $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ lying in the plane (φ, z), then the tangential components take the form

$$\boldsymbol{W} = \begin{pmatrix} \eta_1(r)\boldsymbol{c}_1 \\ \zeta_1(r)\boldsymbol{c}_1 \end{pmatrix} + \begin{pmatrix} \eta_2(r)\boldsymbol{c}_2 \\ \zeta_2(r)\boldsymbol{c}_2 \end{pmatrix}, \tag{8}$$

where planar tensors η_1, ζ_1 equal (planar tensors are defined as $\eta_1 I_r = I_r \eta_1 = \eta_1$)

$$\begin{aligned}
\eta_1 = {} & \vec{e}_1 \vec{w}_1 J_\nu(q_1 r)\boldsymbol{e}_z \otimes \boldsymbol{e}_z + \vec{e}_1 \hat{Z} J_\nu(q_1 r)\vec{w}_1 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z \\
& + \vec{e}_1 \vec{w}_2 J_\nu(q_2 r)\boldsymbol{e}_z \otimes \boldsymbol{e}_\varphi + \vec{e}_1 \hat{Z} J_\nu(q_2 r)\vec{w}_2 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi \\
\zeta_1 = {} & \vec{e}_2 \vec{w}_1 J_\nu(q_1 r)\boldsymbol{e}_z \otimes \boldsymbol{e}_z + \vec{e}_2 \hat{Z} J_\nu(q_1 r)\vec{w}_1 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z \\
& + \vec{e}_2 \vec{w}_2 J_\nu(q_2 r)\boldsymbol{e}_z \otimes \boldsymbol{e}_\varphi + \vec{e}_2 \hat{Z} J_\nu(q_2 r)\vec{w}_2 \boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi.
\end{aligned} \tag{9}$$

Replacing J_ν by Y_ν one can get tensors η_2, ζ_2. In expression (9) we introduce unit two-dimensional vectors $\vec{e}_1 = (1\,0)^T$, $\vec{e}_2 = (0\,1)^T$ and matrix differential operator $\hat{Z}$ equal

$$\hat{Z} = M_{z\varphi}^{(0)-1}\left(\frac{1}{\mathrm{i}k}\frac{\mathrm{d}}{\mathrm{d}r} - \frac{1}{r}M_{zz}^{(1)}\right),$$

where constant matrices $M_{z\varphi}^{(0)}$, $M_{zz}^{(1)}$ can be calculated using the following formulae:

$$M = M^{(0)} + \frac{1}{r}M^{(1)} + \frac{1}{r^2}M^{(2)} \tag{10}$$

$$\begin{aligned} M^{(0)} &= M_{z\varphi}^{(0)}\boldsymbol{e}_z \otimes \boldsymbol{e}_\varphi + M_{\varphi z}^{(0)}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z \\ M^{(1)} &= M_{zz}^{(1)}\boldsymbol{e}_z \otimes \boldsymbol{e}_z + M_{\varphi\varphi}^{(1)}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_\varphi \qquad M^{(2)} = M_{\varphi z}^{(2)}\boldsymbol{e}_\varphi \otimes \boldsymbol{e}_z. \end{aligned} \tag{11}$$

Using such denotations we can rewrite the matrix in Bessel equation (6) as $Q = k^2 M_{z\varphi}^{(0)} M_{\varphi z}^{(0)}$. According to the relationship (9) the tangential field components $\boldsymbol{W}$ include the functions J_ν, $\mathrm{d}J_\nu/\mathrm{d}r$ and J_ν/r, i.e. the Bessel functions determine the solution for cylindrical waves in the medium (2). The waves characterized by η_1, ζ_1 and η_2, ζ_2 correspond to two independent solutions of equations (4) which are expressed by the Bessel functions of the first and second kind, respectively. Tensor notation for cylindrical waves is analogous to the notation for forward and backward plane waves.

Transverse field components $\boldsymbol{H}_\perp$ and $\boldsymbol{E}_\perp$ lying in the plane $z = \text{const}$ are usually introduced for cylindrical beams. These components are continuous in the plane interfaces between media and convenient for the investigation of electromagnetic-beam propagation in multi-layer media. Cylindrical beams are the waves, the amplitudes of which cannot take infinite values in their cross-section. In our case we can use only the waves expressed by the Bessel functions of the first kind with $\eta_1 = \eta$, $\zeta_1 = \zeta$ and $\boldsymbol{c}_1 = \boldsymbol{c}$. Therefore, the tangential field components of the beam have the form

$$\begin{pmatrix}\boldsymbol{H}_\mathrm{t}\\ \boldsymbol{E}_\mathrm{t}\end{pmatrix} = \begin{pmatrix}\eta\boldsymbol{c}\\ \zeta\boldsymbol{c}\end{pmatrix} \equiv \begin{pmatrix}\eta\\ \zeta\end{pmatrix}\boldsymbol{c}.$$

Taking into account the explicit form of the vector $\boldsymbol{c} = c_1\boldsymbol{e}_z + c_2\boldsymbol{e}_\varphi$, the following formula for tangential components can be obtained:

$$\begin{pmatrix}\boldsymbol{H}_\mathrm{t}\\ \boldsymbol{E}_\mathrm{t}\end{pmatrix} = c_1\begin{pmatrix}\eta\boldsymbol{e}_z\\ \zeta\boldsymbol{e}_z\end{pmatrix} + c_2\begin{pmatrix}\eta\boldsymbol{e}_\varphi\\ \zeta\boldsymbol{e}_\varphi\end{pmatrix}. \tag{12}$$

Each of the waves in equation (12) is independent one: at $c_2 = 0$ the first wave propagates and vice versa. These waves correspond to the ordinary and extraordinary waves arising in anisotropic media and are characterized in general case by the different longitudinal wavenumbers β_1 and β_2. Therefore, the field strengths of the cylindrical beam are equal to

$$\begin{pmatrix}\boldsymbol{H}(\boldsymbol{r})\\ \boldsymbol{E}(\boldsymbol{r})\end{pmatrix} = \mathrm{e}^{\mathrm{i}\nu\varphi+\mathrm{i}\beta_1 z}V(\beta_1)\begin{pmatrix}\eta(r,\beta_1)\boldsymbol{e}_z\\ \zeta(r,\beta_1)\boldsymbol{e}_z\end{pmatrix}c_1 + \mathrm{e}^{\mathrm{i}\nu\varphi+\mathrm{i}\beta_2 z}V(\beta_2)\begin{pmatrix}\eta(r,\beta_2)\boldsymbol{e}_\varphi\\ \zeta(r,\beta_2)\boldsymbol{e}_\varphi\end{pmatrix}c_2, \tag{13}$$

where V is the matrix allowing to restore the total field vectors using their tangential components

$$V = \begin{pmatrix} I_r + \boldsymbol{e}_r \otimes \boldsymbol{v}_1 & \boldsymbol{e}_r \otimes \boldsymbol{v}_2\\ \boldsymbol{e}_r \otimes \boldsymbol{v}_3 & I_r + \boldsymbol{e}_r \otimes \boldsymbol{v}_4\end{pmatrix}. \tag{14}$$

Introducing the initial strength vector in the beam cross-section $\boldsymbol{a} = c_1\boldsymbol{e}_r + c_2\boldsymbol{e}_\varphi$ one can represent the fields as

$$\begin{pmatrix}\boldsymbol{H}(\boldsymbol{r})\\ \boldsymbol{E}(\boldsymbol{r})\end{pmatrix} = \mathrm{e}^{\mathrm{i}\nu\varphi}\left[\mathrm{e}^{\mathrm{i}\beta_1 z}V(\beta_1)\begin{pmatrix}\eta(r,\beta_1)\boldsymbol{e}_z\otimes\boldsymbol{e}_r\\ \zeta(r,\beta_1)\boldsymbol{e}_z\otimes\boldsymbol{e}_r\end{pmatrix} + \mathrm{e}^{\mathrm{i}\beta_2 z}V(\beta_2)\begin{pmatrix}\eta(r,\beta_2)\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_\varphi\\ \zeta(r,\beta_2)\boldsymbol{e}_\varphi\otimes\boldsymbol{e}_\varphi\end{pmatrix}\right]\boldsymbol{a}. \tag{15}$$

Then the transverse field components $\boldsymbol{H}_\perp = I_z \boldsymbol{H}$ and $\boldsymbol{E}_\perp = I_z \boldsymbol{E}$ equal

$$\begin{pmatrix} \boldsymbol{H}_\perp(\boldsymbol{r}) \\ \boldsymbol{E}_\perp(\boldsymbol{r}) \end{pmatrix} = \mathrm{e}^{\mathrm{i}\nu\varphi} \begin{pmatrix} \tau(r,z) \\ \sigma(r,z) \end{pmatrix} \boldsymbol{a}, \tag{16}$$

where τ and σ are the planar tensors. Planar tensor τ is defined as $\tau I_z = I_z \tau = \tau$. Tensors τ and σ are of the form

$$\begin{aligned} \tau &= \mathrm{e}^{\mathrm{i}\beta_1 z} \boldsymbol{f}_1(r,\beta_1) \otimes \boldsymbol{e}_r + \mathrm{e}^{\mathrm{i}\beta_2 z} \boldsymbol{f}_2(r,\beta_2) \otimes \boldsymbol{e}_\varphi \\ \sigma &= \mathrm{e}^{\mathrm{i}\beta_1 z} \boldsymbol{g}_1(r,\beta_1) \otimes \boldsymbol{e}_r + \mathrm{e}^{\mathrm{i}\beta_2 z} \boldsymbol{g}_2(r,\beta_2) \otimes \boldsymbol{e}_\varphi. \end{aligned} \tag{17}$$

Vectors $\boldsymbol{f}$ and $\boldsymbol{g}$ equal

$$\begin{aligned} \boldsymbol{f}_1 &= (\boldsymbol{e}_\varphi \eta(r,\beta_1)\boldsymbol{e}_z)\boldsymbol{e}_\varphi + (\boldsymbol{v}_1(\beta_1)\eta(r,\beta_1)\boldsymbol{e}_z + \boldsymbol{v}_2(\beta_1)\zeta(r,\beta_1)\boldsymbol{e}_z)\boldsymbol{e}_r \\ \boldsymbol{f}_2 &= (\boldsymbol{e}_\varphi \eta(r,\beta_2)\boldsymbol{e}_\varphi)\boldsymbol{e}_\varphi + (\boldsymbol{v}_1(\beta_2)\eta(r,\beta_2)\boldsymbol{e}_\varphi + \boldsymbol{v}_2(\beta_2)\zeta(r,\beta_2)\boldsymbol{e}_\varphi)\boldsymbol{e}_r \\ \boldsymbol{g}_1 &= (\boldsymbol{e}_\varphi \zeta(r,\beta_1)\boldsymbol{e}_z)\boldsymbol{e}_\varphi + (\boldsymbol{v}_3(\beta_1)\eta(r,\beta_1)\boldsymbol{e}_z + \boldsymbol{v}_4(\beta_1)\zeta(r,\beta_1)\boldsymbol{e}_z)\boldsymbol{e}_r \\ \boldsymbol{g}_2 &= (\boldsymbol{e}_\varphi \zeta(r,\beta_2)\boldsymbol{e}_\varphi)\boldsymbol{e}_\varphi + (\boldsymbol{v}_3(\beta_2)\eta(r,\beta_2)\boldsymbol{e}_\varphi + \boldsymbol{v}_4(\beta_2)\zeta(r,\beta_2)\boldsymbol{e}_\varphi)\boldsymbol{e}_r. \end{aligned} \tag{18}$$

Formula (16) describes the vector Bessel beam (cylindrical eigenwave) in a bianisotropic medium [7]. Tensors and vectors entered into equation (16) can be written both in Cartesian and cylindrical coordinates. In Cartesian coordinates cylindrical basis vectors $\boldsymbol{e}_r, \boldsymbol{e}_\varphi, \boldsymbol{e}_z$ depend on azimuthal coordinate φ, while Cartesian basis vectors are constant. In cylindrical coordinates the azimuthal angle enters into Cartesian basis vectors. Further in this paper, we will use cylindrical coordinates.

Tensor notation (16) follows directly from solution of Maxwell's equations. In such notation the coordinate dependence of the fields is separated from constant vector $\boldsymbol{a}$ determined by the initial conditions. It is the most important result of this section, because the constructing of an arbitrary vector beam is based on this separation.

One can note that r dependence in each vector $\boldsymbol{f}$ and $\boldsymbol{g}$ is expressed by the functions $\mathrm{d}J_\nu/\mathrm{d}r$ and J_ν/r, which can be written by means of the Bessel functions of the orders $\nu - 1$ and $\nu + 1$. That is why the r dependence of the tensors τ and σ takes the form

$$\tau = J_{\nu-1}(qr)\tau^-(z) + J_{\nu+1}(qr)\tau^+(z), \qquad \sigma = J_{\nu-1}(qr)\sigma^-(z) + J_{\nu+1}(qr)\sigma^+(z). \tag{19}$$

Generally, tensors $\tau^\pm$ and $\sigma^\pm$ should be considered as planar. However, these tensors are dyads in examples calculated in section 4. Further we suppose that tensors $\tau^\pm$ and $\sigma^\pm$ are dyads. For dyads $\tau^\pm$ and $\sigma^\pm$ one can always choose such initial vector $\boldsymbol{a}$ which makes vanish one of the dyads. Therefore, the beams described by the Bessel functions of the orders $\nu - 1$ and $\nu + 1$ are independent.

Taking into account z dependence we represent tensors $\tau^\pm$ and $\sigma^\pm$ as follows:

$$\tau^\pm = \boldsymbol{p}^\pm \otimes \left(\mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{p}_1^\pm + \mathrm{e}^{\mathrm{i}\beta_2 z}\boldsymbol{p}_2^\pm\right), \qquad \sigma^\pm = \boldsymbol{s}^\pm \otimes \left(\mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{s}_1^\pm + \mathrm{e}^{\mathrm{i}\beta_2 z}\boldsymbol{s}_2^\pm\right), \tag{20}$$

where $\boldsymbol{p}^\pm, \boldsymbol{p}_{1,2}^\pm, \boldsymbol{s}^\pm, \boldsymbol{s}_{1,2}^\pm$ are q-dependent vectors.

3. Constructing of an arbitrary beam using cylindrical partial waves

Formula (16) describes the family of vector Bessel beams with different azimuthal numbers ν and transverse (radial) wavenumbers q. The numbers q enter into tensors η and ζ, as well as into longitudinal wavenumbers $\beta_{1,2}$. For example, in an isotropic medium longitudinal wavenumbers equal $\beta_1 = \beta_2 = \sqrt{k^2\varepsilon\mu - q^2}$. Transverse wavenumbers are the same for both independent waves. An arbitrary vector function can be written as the superposition of cylindrical partial waves:

$$\begin{pmatrix} \boldsymbol{H}_\perp(r,\varphi,z) \\ \boldsymbol{E}_\perp(r,\varphi,z) \end{pmatrix} = \sum_{\nu=-\infty}^{\infty} \int_0^\infty \mathrm{e}^{\mathrm{i}\nu\varphi} \begin{pmatrix} \tau_\nu(r,z,q) \\ \sigma_\nu(r,z,q) \end{pmatrix} \boldsymbol{a}(q,\nu) q\,\mathrm{d}q. \tag{21}$$

Vector $\boldsymbol{a}(q,\nu)$ can be expressed by means of a known electric field determined in the initial plane $z=0$. Of course, magnetic field can be initial field, too. From equation (21) it follows the values of electric field in the plane $z=0$:

$$\boldsymbol{E}_\perp(r,\varphi,0) = \sum_{\nu=-\infty}^{\infty} \int_0^\infty \mathrm{e}^{\mathrm{i}\nu\varphi} \sigma_\nu(r,0,q)\boldsymbol{a}(q,\nu) q\,\mathrm{d}q. \tag{22}$$

Integrating over φ we sum up over ν as follows:

$$\boldsymbol{E}_\perp(r,\nu,0) = \int_0^\infty \mathrm{e}^{\mathrm{i}\nu\varphi} \sigma_\nu(r,0,q)\boldsymbol{a}(q,\nu) q\,\mathrm{d}q, \tag{23}$$

where

$$\boldsymbol{E}_\perp(r,\nu,0) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{e}^{-\mathrm{i}\nu\varphi'} \boldsymbol{E}_\perp(r,\varphi',0)\,\mathrm{d}\varphi'.$$

Equation (23) describes tensor link between electric field vector $\boldsymbol{E}_\perp(r,\nu,0)$ and its Fourier transform $\boldsymbol{a}(q,\nu)$. This link can be called tensor Fourier transform. Inverse tensor Fourier transform is determined by tensor Θ_ν:

$$\boldsymbol{a}(q,\nu) = \int_0^\infty \Theta_\nu(r',0,q)\boldsymbol{E}_\perp(r',\nu,0) r'\,\mathrm{d}r'. \tag{24}$$

By substituting expression (24) into (23) we obtain equation

$$\boldsymbol{E}_\perp(r,\nu,0) = \int_0^\infty q\,\mathrm{d}q \int_0^\infty r'\,\mathrm{d}r' \sigma_\nu(r,0,q)\Theta_\nu(r',0,q)\boldsymbol{E}_\perp(r',\nu,0) \tag{25}$$

from which planar tensor Θ_ν should be found. Expression (25) is analogous to the well-known Fourier–Bessel transform [24]

$$f(r) = \int_0^\infty q\,\mathrm{d}q \int_0^\infty r'\,\mathrm{d}r' J_\nu(qr)J_\nu(qr')f(r'). \tag{26}$$

Since tensor σ is determined by formulae (19) and (20), one can find tensor Θ as follows:

$$\Theta_\nu(r',0,q) = J_{\nu-1}(qr')\frac{\boldsymbol{e}_z^\times\left(\boldsymbol{s}_1^+ + \boldsymbol{s}_2^+\right)\otimes \boldsymbol{s}^+\boldsymbol{e}_z^\times}{N_1N_2} + J_{\nu+1}(qr')\frac{\boldsymbol{e}_z^\times\left(\boldsymbol{s}_1^- + \boldsymbol{s}_2^-\right)\otimes \boldsymbol{s}^-\boldsymbol{e}_z^\times}{N_1N_2}, \tag{27}$$

where $N_1 = \left(\boldsymbol{s}_1^- + \boldsymbol{s}_2^-\right)\boldsymbol{e}_z^\times\left(\boldsymbol{s}_1^+ + \boldsymbol{s}_2^+\right)$, $N_2 = \boldsymbol{s}^+\boldsymbol{e}_z^\times\boldsymbol{s}^-$ are normalization coefficients. The product of tensors is equal to

$$\sigma_\nu(r,0,q)\Theta_\nu(r',0,q) = J_{\nu-1}(qr)J_{\nu-1}(qr')\frac{\boldsymbol{s}^-\otimes\boldsymbol{s}^+\boldsymbol{e}_z^\times}{N_2} + J_{\nu+1}(qr)J_{\nu+1}(qr')\frac{\boldsymbol{s}^+\otimes\boldsymbol{e}_z^\times\boldsymbol{s}^-}{N_2}. \tag{28}$$

By substituting (28) into (25) and integrating over q and r' we get to the identity

$$\boldsymbol{E}_\perp(r,\nu,0) = \left(\frac{\boldsymbol{s}^-\otimes\boldsymbol{s}^+\boldsymbol{e}_z^\times}{N_2} + \frac{\boldsymbol{s}^+\otimes\boldsymbol{e}_z^\times\boldsymbol{s}^-}{N_2}\right)\boldsymbol{E}_\perp(r,\nu,0) \equiv I_z\boldsymbol{E}_\perp(r,\nu,0). \tag{29}$$

Hence, tensor Θ is derived correctly, and tensor Fourier transform (25) is satisfied. It should be noted that only for dyads $\tau^\pm$ and $\sigma^\pm$, one can find the inverse tensor Fourier transform, because only in this case the product (28) can be written without cross terms containing the products of the Bessel functions $J_{\nu\mp1}(qr)J_{\nu\pm1}(qr')$. That is indirect confirmation that tensors $\tau^{\pm}$ and $\sigma^{\pm}$ are dyads for all media of the form (2).

By substituting tensor Θ into equation (21) we derive the following general formula for transverse electric field strength of an arbitrary beam in a bianisotropic medium:

$$\begin{aligned}\boldsymbol{E}_{\perp}(r,\varphi,z) = \frac{1}{2\pi}\sum_{\nu=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\nu\varphi}\int_0^{\infty} q\,\mathrm{d}q\int_0^{\infty} r'\,\mathrm{d}r'\sum_{j=1,2}\mathrm{e}^{\mathrm{i}\beta_j(q)z}\big(J_{\nu-1}(qr)J_{\nu-1}(qr')\rho_j^-(q)\\ +(-1)^{j-1}J_{\nu-1}(qr)J_{\nu+1}(qr')\rho^-(q)+(-1)^{j-1}J_{\nu+1}(qr)J_{\nu-1}(qr')\rho^+(q)\\ +J_{\nu+1}(qr)J_{\nu+1}(qr')\rho_j^+(q)\big)\int_0^{2\pi}\mathrm{d}\varphi'\,\mathrm{e}^{-\mathrm{i}\nu\varphi'}\boldsymbol{E}_{\perp}(r',\varphi',0),\end{aligned} \tag{30}$$

where dyads ρ depend on transverse wavenumber and equal

$$\rho_j^{\pm} = \frac{\boldsymbol{s}_j^{\pm}\boldsymbol{e}_z^{\times}\left(\boldsymbol{s}_1^{\mp}+\boldsymbol{s}_2^{\mp}\right)}{N_1N_2}\boldsymbol{s}^{\pm}\otimes\boldsymbol{s}^{\mp}\boldsymbol{e}_z^{\times}, \qquad \rho^{\pm} = \frac{\boldsymbol{s}_1^{\pm}\boldsymbol{e}_z^{\times}\boldsymbol{s}_2^{\pm}}{N_1N_2}\boldsymbol{s}^{\pm}\otimes\boldsymbol{s}^{\pm}\boldsymbol{e}_z^{\times}. \tag{31}$$

Index j enumerates the eigenwaves in a bianisotropic medium. Formula (30) expresses an exact wave evolution for given field strengths in beam cross-section $z=0$. Calculating the product $\tau(r,z,q)\Theta(r',0,q)$ in equation (21) the magnetic field can be found. It is obvious that the formula for magnetic field is more cumbersome than that for electric field.

In paraxial approximation the transverse wavenumber q is supposed to be much less than wavenumber k. Since numbers q^2 are the eigenvalues of matrix Q in equation for longitudinal field components (6), the longitudinal wavenumber β depends on q^2. The propagation constant β can be expanded into series as $\beta(q)\approx\beta(0)-q^2/\widetilde{\beta}$. In paraxial propagation all tensors ρ are constant: $\rho(q)\approx\rho(0)=\text{const}$. Then, from formula (30) we get to expression for the beam electric field

$$\begin{aligned}\boldsymbol{E}_{\perp}(r,\varphi,z) = \sum_{\nu=-\infty}^{\infty}\mathrm{e}^{\mathrm{i}\nu\varphi}\int_0^{\infty} r'\,\mathrm{d}r'\sum_{j=1,2}\mathrm{e}^{\mathrm{i}\beta_j(0)z}\\ \times\left[\frac{\widetilde{\beta}_j}{2\mathrm{i}z}\exp\left(\mathrm{i}\widetilde{\beta}_j\frac{r^2+r'^2}{4z}\right)\left(I_{\nu-1}\left(\frac{\widetilde{\beta}_jrr'}{2\mathrm{i}z}\right)\rho_j^-(0)+I_{\nu+1}\left(\frac{\widetilde{\beta}_jrr'}{2\mathrm{i}z}\right)\rho_j^+(0)\right)\right.\\ \left.+(-1)^{j-1}(F_j(r,r',z)\rho^-(0)+F_j(r',r,z)\rho^+(0))\right]\boldsymbol{E}_{\perp}(r',\nu,0).\end{aligned} \tag{32}$$

Here we use the following formula:

$$\int_0^{\infty} q\,\mathrm{d}q\exp(-\alpha q^2)J_{\nu}(\gamma q)J_{\nu}(\delta q) = \frac{1}{2\alpha}\exp\left(-\frac{\gamma^2+\delta^2}{4\alpha}\right)I_{\nu}\left(\frac{\gamma\delta}{2\alpha}\right) \tag{33}$$

and designation

$$F_j(r,r',z) = \int_0^{\infty} q\,\mathrm{d}q\exp(-\mathrm{i}q^2z/\widetilde{\beta}_j)J_{\nu-1}(qr)J_{\nu+1}(qr'), \tag{34}$$

where I_{ν} is the modified Bessel function. Paraxial wave is described by the complex formula, but expression (32) is simpler than an exact solution (30), because integration over q is eliminated.

4. Electromagnetic beams in complex media

We will not focus our attention on the computation of tensors τ and σ. They can be found by means of formulae of section 2 (see the paper [7] as well). Tensors τ and σ are given for each media at once. In this investigation we emphasize the beam constructing using partial wave superposition. To compare the results of beam propagation in different complex media we choose the following cylindrically symmetric beam polarized in x direction:

$$\boldsymbol{E}_{\perp}(r, \varphi, 0) = \mathrm{e}^{\mathrm{i}m\varphi} f(r)\boldsymbol{e}_x. \tag{35}$$

Choosing the function $f(r)$ one can obtain field distributions for Bessel–Gauss beams, Hermite–Gauss beams, Laguerre–Gauss beams, etc. Cartesian basis vector $\boldsymbol{e}_x$ should be written in cylindrical coordinates. It is convenient to express it using circular vectors $\boldsymbol{e}_{\pm}$ as follows:

$$\boldsymbol{e}_x = \tfrac{1}{\sqrt{2}}(\mathrm{e}^{\mathrm{i}\varphi}\boldsymbol{e}_+ + \mathrm{e}^{-\mathrm{i}\varphi}\boldsymbol{e}_-), \qquad \boldsymbol{e}_{\pm} = \tfrac{1}{\sqrt{2}}(\boldsymbol{e}_r \pm \mathrm{i}\boldsymbol{e}_{\varphi}).$$

The main properties of circular vectors are $\boldsymbol{e}_{\pm}^2 = 0, \boldsymbol{e}_+\boldsymbol{e}_- = 1$. Hence, the Fourier transform of the initial electric field distribution equals

$$\boldsymbol{E}_{\perp}(r, \nu, 0) = \frac{f(r)}{\sqrt{2}}(\delta_{\nu,m+1}\boldsymbol{e}_+ + \delta_{\nu,m-1}\boldsymbol{e}_-). \tag{36}$$

4.1. Isotropic medium

An isotropic medium is characterized by the scalar values of dielectric permittivity ε and magnetic permeability μ. Tensors τ and σ are equal to

$$\begin{aligned}\tau(r, q) &= \frac{\exp(\mathrm{i}\beta z)}{\sqrt{2}q}(J_{\nu-1}(qr)\boldsymbol{e}_+ \otimes (\mathrm{i}\beta\boldsymbol{e}_r + k\varepsilon\boldsymbol{e}_{\varphi}) + J_{\nu+1}(qr)\boldsymbol{e}_- \otimes (-\mathrm{i}\beta\boldsymbol{e}_r + k\varepsilon\boldsymbol{e}_{\varphi}))\\ \sigma(r, q) &= \frac{\exp(\mathrm{i}\beta z)}{\sqrt{2}q}(J_{\nu-1}(qr)\boldsymbol{e}_+ \otimes (\mathrm{i}\beta\boldsymbol{e}_{\varphi} - k\mu\boldsymbol{e}_r) - J_{\nu+1}(qr)\boldsymbol{e}_- \otimes (\mathrm{i}\beta\boldsymbol{e}_{\varphi} + k\mu\boldsymbol{e}_r)),\end{aligned} \tag{37}$$

where $\beta(q) = \sqrt{k^2\varepsilon\mu - q^2}$. In the isotropic medium both eigenwaves coincide: $\beta_1 = \beta_2 = \beta$. Comparing expression (37) with (19), (20), we note that tensors $\tau_{\pm}$ and $\sigma_{\pm}$ are dyads composed of the vectors

$$\boldsymbol{p}^{\pm} = \boldsymbol{e}_{\mp}, \qquad \boldsymbol{p}_1^{\pm} + \boldsymbol{p}_2^{\pm} = \frac{\mp\mathrm{i}\beta\boldsymbol{e}_r + k\varepsilon\boldsymbol{e}_{\varphi}}{\sqrt{2}q}, \qquad \boldsymbol{s}^{\pm} = \boldsymbol{e}_{\mp}, \qquad \boldsymbol{s}_1^{\pm} + \boldsymbol{s}_2^{\pm} = \frac{\mp\mathrm{i}\beta\boldsymbol{e}_{\varphi} - k\mu\boldsymbol{e}_r}{\sqrt{2}q}.$$

According to equation (27) tensor Θ determining the inverse Fourier transform is of the form

$$\Theta_{\nu}(r', 0, q) = \frac{\mathrm{i}q}{\sqrt{2}k\mu\beta}(J_{\nu-1}(qr')(\mathrm{i}\beta\boldsymbol{e}_r - k\mu\boldsymbol{e}_{\varphi}) \otimes \boldsymbol{e}_- + J_{\nu+1}(qr')(\mathrm{i}\beta\boldsymbol{e}_r + k\mu\boldsymbol{e}_{\varphi}) \otimes \boldsymbol{e}_+). \tag{38}$$

Tensors ρ entered into formula (30) are constant and equal to

$$\rho_{1,2}^{+} = \tfrac{1}{2}\boldsymbol{e}_- \otimes \boldsymbol{e}_+, \qquad \rho_{1,2}^{-} = \tfrac{1}{2}\boldsymbol{e}_+ \otimes \boldsymbol{e}_-, \qquad \rho^{+} = \tfrac{1}{2}\boldsymbol{e}_- \otimes \boldsymbol{e}_-, \qquad \rho^{-} = -\tfrac{1}{2}\boldsymbol{e}_+ \otimes \boldsymbol{e}_+. \tag{39}$$

Therefore, the electric field of the vector beam varies during its propagation as follows:

$$\begin{aligned}\boldsymbol{E}_{\perp}(r, \varphi, z) = \sum_{\nu=-\infty}^{\infty}\int_0^{\infty} q\,\mathrm{d}q \int_0^{\infty} r'\,\mathrm{d}r'\,\mathrm{e}^{\mathrm{i}\beta(q)z+\mathrm{i}\nu\varphi}(J_{\nu-1}(qr)J_{\nu-1}(qr')\boldsymbol{e}_+ \otimes \boldsymbol{e}_-\\ + J_{\nu+1}(qr)J_{\nu+1}(qr')\boldsymbol{e}_- \otimes \boldsymbol{e}_+)\boldsymbol{E}_{\perp}(r', \nu, 0).\end{aligned} \tag{40}$$

Electric field (40) is written as the superposition of left and right circularly polarized beams. In such a way an arbitrary polarized vector beam can be described. The field strength

(40) does not include the terms characterized by the products of the Bessel functions of the orders $\nu - 1$ and $\nu + 1$. Magnetic field is expressed by significantly more complex formula

$$\boldsymbol{H}_\perp(r,\varphi,z) = \sum_{\nu=-\infty}^{\infty} \int_0^\infty q\,\mathrm{d}q \int_0^\infty r'\,\mathrm{d}r'\,\mathrm{e}^{\mathrm{i}\beta(q)z+\mathrm{i}\nu\varphi}\left[\frac{2k^2\varepsilon\mu - q^2}{2\mathrm{i}k\mu\beta(q)}(J_{\nu-1}(qr)J_{\nu-1}(qr')\boldsymbol{e}_+\otimes\boldsymbol{e}_- - J_{\nu+1}(qr)J_{\nu+1}(qr')\boldsymbol{e}_-\otimes\boldsymbol{e}_+) + \frac{q^2}{2\mathrm{i}k\mu\beta(q)}(J_{\nu+1}(qr)J_{\nu-1}(qr')\boldsymbol{e}_-\otimes\boldsymbol{e}_- - J_{\nu-1}(qr)J_{\nu+1}(qr')\boldsymbol{e}_+\otimes\boldsymbol{e}_+)\right]\boldsymbol{E}_\perp(r',\nu,0). \tag{41}$$

Note the existence of the functions $J_{\nu\pm1}(qr)J_{\nu\mp1}(qr')$ in the exact formula for the magnetic field. They vanish in paraxial approximation. Paraxial solutions for the electric and magnetic fields take the form

$$\boldsymbol{E}_\perp(r,\varphi,z) = -\frac{\mathrm{i}k\sqrt{\varepsilon\mu}}{z}\,\mathrm{e}^{\mathrm{i}k\sqrt{\varepsilon\mu}(z+r^2/z)}\sum_{\nu=-\infty}^{\infty}\int_0^\infty r'\,\mathrm{d}r'\,\mathrm{e}^{\mathrm{i}k\sqrt{\varepsilon\mu}r'^2/z+\mathrm{i}\nu\varphi} \times\left(I_{\nu-1}\left(-\mathrm{i}k\sqrt{\varepsilon\mu}\frac{rr'}{z}\right)\boldsymbol{e}_+\otimes\boldsymbol{e}_- + I_{\nu+1}\left(-\mathrm{i}k\sqrt{\varepsilon\mu}\frac{rr'}{z}\right)\boldsymbol{e}_-\otimes\boldsymbol{e}_+\right)\boldsymbol{E}_\perp(r',\nu,0) \tag{42}$$

$$\boldsymbol{H}_\perp(r,\varphi,z) = -\frac{k\varepsilon}{z}\,\mathrm{e}^{\mathrm{i}k\sqrt{\varepsilon\mu}(z+r^2/z)}\sum_{\nu=-\infty}^{\infty}\int_0^\infty r'\,\mathrm{d}r'\,\mathrm{e}^{\mathrm{i}k\sqrt{\varepsilon\mu}r'^2/z+\mathrm{i}\nu\varphi} \times\left(I_{\nu-1}\left(-\mathrm{i}k\sqrt{\varepsilon\mu}\frac{rr'}{z}\right)\boldsymbol{e}_+\otimes\boldsymbol{e}_- - I_{\nu+1}\left(-\mathrm{i}k\sqrt{\varepsilon\mu}\frac{rr'}{z}\right)\boldsymbol{e}_-\otimes\boldsymbol{e}_+\right)\boldsymbol{E}_\perp(r',\nu,0). \tag{43}$$

One can make sure that in paraxial approximation electric and magnetic field strengths are connected with each other by the relationship $\boldsymbol{E}_\perp = -\sqrt{\varepsilon/\mu}(\boldsymbol{e}_z \times \boldsymbol{H}_\perp)$, i.e. the constant quantity $\sqrt{\varepsilon/\mu}$ is the impedance as for the plane waves in an isotropic medium.

Let us consider a cylindrically symmetric beam (35). By substituting such initial electric field distribution into equation (40) we obtain

$$\boldsymbol{E}_\perp(r,\varphi,z) = \mathrm{e}^{\mathrm{i}m\varphi}g_m(r,z)\boldsymbol{e}_x, \tag{44}$$

where

$$g_m(r,z) = \int_0^\infty q\,\mathrm{d}q\int_0^\infty r'\,\mathrm{d}r'\,\mathrm{e}^{\mathrm{i}\beta(q)z}J_m(qr)J_m(qr')f(r'). \tag{45}$$

In paraxial approximation the function $g_m(r,z)$ takes the form

$$g_m(r,z) = \frac{k\sqrt{\varepsilon\mu}}{\mathrm{i}z}\,\mathrm{e}^{\mathrm{i}k\sqrt{\varepsilon\mu}z}\int_0^\infty r'\,\mathrm{d}r'\exp\left(\mathrm{i}k\sqrt{\varepsilon\mu}\frac{r^2+r'^2}{2z}\right)I_{\nu-1}\left(\frac{k\sqrt{\varepsilon\mu}rr'}{\mathrm{i}z}\right)f(r'). \tag{46}$$

Since the polarization of cylindrically symmetric beam does not change in an isotropic medium, this electromagnetic beam can be considered as scalar one, while its evolution is described using $g_m(r,z)$.

4.2. Bi-isotropic medium

Let a bi-isotropic medium is characterized by the scalar values of dielectric permittivity ε, magnetic permeability μ and gyration parameters $\alpha = \mathrm{i}\chi$, $\beta = -\mathrm{i}\chi$. In such a medium two eigenwaves propagate. They have different longitudinal wavenumbers

$$\beta_1(q) = \sqrt{k^2(\sqrt{\varepsilon\mu} - \chi)^2 - q^2}, \qquad \beta_2(q) = \sqrt{k^2(\sqrt{\varepsilon\mu} + \chi)^2 - q^2}.$$

Tensors τ and σ equal

$$\tau = \frac{1}{q}\sqrt{\frac{\varepsilon}{\mu}}(\mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{b}_1 \otimes \boldsymbol{e}_r + \mathrm{e}^{\mathrm{i}\beta_2 z}\boldsymbol{b}_2 \otimes \boldsymbol{e}_\varphi) \qquad \sigma = \frac{\mathrm{i}}{q}(-\mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{b}_1 \otimes \boldsymbol{e}_r + \mathrm{e}^{\mathrm{i}\beta_2 z}\boldsymbol{b}_2 \otimes \boldsymbol{e}_\varphi), \tag{47}$$

where

$$\boldsymbol{b}_1 = \tfrac{1}{\sqrt{2}}(k(\sqrt{\varepsilon\mu} - \chi) - \beta_1)J_{\nu-1}(qr)\boldsymbol{e}_+ + \tfrac{1}{\sqrt{2}}(k(\sqrt{\varepsilon\mu} - \chi) + \beta_1)J_{\nu+1}(qr)\boldsymbol{e}_-$$
$$\boldsymbol{b}_2 = \tfrac{1}{\sqrt{2}}(k(\sqrt{\varepsilon\mu} + \chi) + \beta_2)J_{\nu-1}(qr)\boldsymbol{e}_+ + \tfrac{1}{\sqrt{2}}(k(\sqrt{\varepsilon\mu} + \chi) - \beta_2)J_{\nu+1}(qr)\boldsymbol{e}_-.$$

Therefore, vectors $\boldsymbol{p}$ and $\boldsymbol{s}$ depend on q in a complicated way. They can be written as follows:

$$\boldsymbol{p}^\pm = \boldsymbol{e}_\mp, \qquad \boldsymbol{p}_1^\pm = \frac{1}{q}\sqrt{\frac{\varepsilon}{2\mu}}(k(\sqrt{\varepsilon\mu} - \chi) \pm \beta_1)\boldsymbol{e}_r, \qquad \boldsymbol{p}_2^\pm = \frac{1}{q}\sqrt{\frac{\varepsilon}{2\mu}}(k(\sqrt{\varepsilon\mu} + \chi) \mp \beta_2)\boldsymbol{e}_\varphi$$
$$\boldsymbol{s}^\pm = \boldsymbol{e}_\mp, \qquad \boldsymbol{s}_1^\pm = -\frac{\mathrm{i}}{\sqrt{2}q}(k(\sqrt{\varepsilon\mu} - \chi) \pm \beta_1)\boldsymbol{e}_r, \qquad \boldsymbol{s}_2^\pm = \frac{\mathrm{i}}{\sqrt{2}q}(k(\sqrt{\varepsilon\mu} + \chi) \mp \beta_2)\boldsymbol{e}_\varphi. \tag{48}$$

Tensors ρ are calculated using formulae (31):

$$\begin{aligned} &\rho_1^+ = a_{11}\boldsymbol{e}_- \otimes \boldsymbol{e}_+, && \rho_2^+ = -a_{22}\boldsymbol{e}_- \otimes \boldsymbol{e}_+, && \rho_1^- = -a_{22}\boldsymbol{e}_+ \otimes \boldsymbol{e}_-, \\ &\rho_2^- = a_{11}\boldsymbol{e}_+ \otimes \boldsymbol{e}_-, && \rho^+ = -a_{12}\boldsymbol{e}_- \otimes \boldsymbol{e}_-, && \rho^- = a_{21}\boldsymbol{e}_+ \otimes \boldsymbol{e}_+, \end{aligned} \tag{49}$$

where

$$a_{ij}(q) = \frac{(k(\sqrt{\varepsilon\mu} - \chi) + (-1)^{i-1}\beta_1)(k(\sqrt{\varepsilon\mu} + \chi) + (-1)^{j-1}\beta_2)}{2(k(\sqrt{\varepsilon\mu} + \chi)\beta_1 + k(\sqrt{\varepsilon\mu} - \chi)\beta_2)}.$$

We do not adduce the exact formula for the electric field, because its form is not simpler than the general expression (30). In paraxial approximation an arbitrary vector beam in a bi-isotropic medium is described by the following parameters:

$$\begin{aligned} &\beta_1(0) = k(\sqrt{\varepsilon\mu} - \chi), && \beta_2(0) = k(\sqrt{\varepsilon\mu} + \chi), && \widetilde{\beta}_{1,2} = 2\beta_{1,2} \\ &a_{11}(0) = 1, && a_{12}(0) = a_{21}(0) = a_{22}(0) = 0. \end{aligned}$$

So, the paraxial beam propagates as follows:

$$\begin{aligned} \boldsymbol{E}_\perp(r, \varphi, z) = \mathrm{e}^{\mathrm{i}k\sqrt{\varepsilon\mu}z} \sum_{\nu=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\nu\varphi} \int_0^\infty r'\,\mathrm{d}r' \Bigg[&\frac{k(\sqrt{\varepsilon\mu} + \chi)}{\mathrm{i}z} \\ &\times \exp\left(\mathrm{i}k\chi z + \mathrm{i}k(\sqrt{\varepsilon\mu} + \chi)\frac{r^2 + r'^2}{2z}\right) I_{\nu-1}\left(\frac{k(\sqrt{\varepsilon\mu} + \chi)rr'}{\mathrm{i}z}\right) \boldsymbol{e}_+ \otimes \boldsymbol{e}_- \\ &+ \frac{k(\sqrt{\varepsilon\mu} - \chi)}{\mathrm{i}z} \exp\left(-\mathrm{i}k\chi z + \mathrm{i}k(\sqrt{\varepsilon\mu} - \chi)\frac{r^2 + r'^2}{2z}\right) \\ &\times I_{\nu+1}\left(\frac{k(\sqrt{\varepsilon\mu} - \chi)rr'}{\mathrm{i}z}\right) \boldsymbol{e}_- \otimes \boldsymbol{e}_+ \Bigg] \boldsymbol{E}_\perp(r', \nu, 0). \end{aligned} \tag{50}$$

The paraxial solution represents the superposition of two eigenwaves, the eigenwaves being right and left polarized. Applying formula (50) for cylindrically symmetric initial field distribution (35) we obtain the electric strength vector

$$\boldsymbol{E}_\perp(r, \varphi, z) = \tfrac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}m\varphi}\left(g_m^{(1)}\boldsymbol{e}_-\mathrm{e}^{-\mathrm{i}\varphi} + g_m^{(2)}\boldsymbol{e}_+\mathrm{e}^{\mathrm{i}\varphi}\right), \tag{51}$$

where the functions $g_m^{(1,2)}$ follow from definition (45), in which we should substitute $\beta_{1,2}$ for β. In paraxial approximation the functions $g_m^{(1)}$ and $g_m^{(2)}$ are expressed by formula (46), in which the quantity $\sqrt{\varepsilon\mu}$ should be replaced by $\sqrt{\varepsilon\mu} + \chi$ and $\sqrt{\varepsilon\mu} - \chi$, respectively.

4.3. Bianisotropic medium

We consider a bianisotropic medium characterized by the following tensors:

$$\varepsilon = \varepsilon_1 I_z + \varepsilon_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_z, \qquad \mu = \mu_1 I_z + \mu_2 \boldsymbol{e}_z \otimes \boldsymbol{e}_z, \qquad \alpha = \kappa = \mathrm{i}\chi \boldsymbol{e}_z^{\times}. \tag{52}$$

A bianisotropic medium (52) can be applied for the description of the crystals of symmetry classes 3m, 4 mm and 6 mm as in [25]. The waves propagate in the direction of an optic axis of such crystals. In another interpretation constitutive equations (52) correspond to a moving medium, velocity of which is directed along z-axis [26].

Planar tensors τ and σ take the form

$$\begin{aligned}\tau &= \frac{\mu_2(\beta_1 - \mathrm{i}k\chi)}{\mu_1 q}\,\mathrm{e}^{\mathrm{i}\beta_1 z}\boldsymbol{b}\otimes\boldsymbol{e}_r + \frac{k\varepsilon_2}{q}\,\mathrm{e}^{\mathrm{i}\beta_2 z}(\boldsymbol{e}_z\times\boldsymbol{b})\otimes\boldsymbol{e}_\varphi\\ \sigma &= -\frac{k\mu_2}{q}\,\mathrm{e}^{\mathrm{i}\beta_1 z}(\boldsymbol{e}_z\times\boldsymbol{b})\otimes\boldsymbol{e}_r + \frac{\varepsilon_2(\beta_2+\mathrm{i}k\chi)}{\varepsilon_1 q}\,\mathrm{e}^{\mathrm{i}\beta_2 z}\boldsymbol{b}\otimes\boldsymbol{e}_\varphi,\end{aligned} \tag{53}$$

where vector $\boldsymbol{b}$ and wavevector longitudinal components for each eigenwave are written as

$$\begin{aligned}&\boldsymbol{b} = \frac{\mathrm{i}}{\sqrt{2}}\,(J_{\nu-1}\boldsymbol{e}_+ - J_{\nu+1}\boldsymbol{e}_-)\\ &\beta_1 = \sqrt{k^2\varepsilon_1\mu_1 - k^2\chi^2 - q^2\mu_1/\mu_2}, \qquad \beta_2 = \sqrt{k^2\varepsilon_1\mu_1 - k^2\chi^2 - q^2\varepsilon_1/\varepsilon_2}.\end{aligned} \tag{54}$$

Then, vectors $\boldsymbol{p}$ and $\boldsymbol{s}$ are of the form

$$\begin{aligned}&\boldsymbol{p}^{\pm} = \boldsymbol{e}_{\mp}, \qquad \boldsymbol{p}_1^{\pm} = \mp\frac{\mathrm{i}\mu_2(\beta_1 - \mathrm{i}k\chi)}{\sqrt{2}\mu_1 q}\boldsymbol{e}_r, \qquad \boldsymbol{p}_2^{\pm} = \frac{k\varepsilon_2}{\sqrt{2}q}\boldsymbol{e}_\varphi\\ &\boldsymbol{s}^{\pm} = \boldsymbol{e}_{\mp}, \qquad \boldsymbol{s}_1^{\pm} = -\frac{k\mu_2}{\sqrt{2}q}\boldsymbol{e}_r, \qquad \boldsymbol{s}_2^{\pm} = \mp\frac{\mathrm{i}\varepsilon_2(\beta_2+\mathrm{i}k\chi)}{\sqrt{2}\varepsilon_1 q}\boldsymbol{e}_\varphi.\end{aligned} \tag{55}$$

Tensors ρ for a bianisotropic medium under consideration have the same form (39) as that for an isotropic medium. The field evolution is described by the following formula:

$$\begin{aligned}\boldsymbol{E}_\perp(r,\varphi,z) = \sum_{\nu=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\nu\varphi}\int_0^\infty q\,\mathrm{d}q \int_0^\infty r'\,\mathrm{d}r'\Big[\frac{1}{2}(\mathrm{e}^{\mathrm{i}\beta_1(q)z} + \mathrm{e}^{\mathrm{i}\beta_2(q)z})(J_{\nu-1}(qr)J_{\nu-1}(qr')\boldsymbol{e}_+\otimes\boldsymbol{e}_-\\ + J_{\nu+1}(qr)J_{\nu+1}(qr')\boldsymbol{e}_-\otimes\boldsymbol{e}_+) + \frac{1}{2}(\mathrm{e}^{\mathrm{i}\beta_1(q)z} - \mathrm{e}^{\mathrm{i}\beta_2(q)z})(-J_{\nu-1}(qr)\\ \times J_{\nu+1}(qr')\boldsymbol{e}_+\otimes\boldsymbol{e}_+ + J_{\nu+1}(qr)J_{\nu-1}(qr')\boldsymbol{e}_-\otimes\boldsymbol{e}_-)\Big]\boldsymbol{E}_\perp(r',\nu,0).\end{aligned} \tag{56}$$

It is obvious that at $\beta_1 = \beta_2$ one can get to expression (40). By substituting the cylindrically symmetric field distribution (35) into (56) we find the beam evolution

$$\boldsymbol{E}_\perp(r,\varphi,z) = \tfrac{1}{2}\,\mathrm{e}^{\mathrm{i}m\varphi}\big[\big(g_m^{(1)} + g_m^{(2)}\big)\boldsymbol{e}_x + \tfrac{1}{\sqrt{2}}\big(g_m^+\boldsymbol{e}_-\mathrm{e}^{\mathrm{i}\varphi} + g_m^-\boldsymbol{e}_+\mathrm{e}^{-\mathrm{i}\varphi}\big)\big], \tag{57}$$

where

$$g_m^{\pm} = \int_0^\infty q\,\mathrm{d}q\int_0^\infty r'\,\mathrm{d}r'(\mathrm{e}^{\mathrm{i}\beta_1(q)z} - \mathrm{e}^{\mathrm{i}\beta_2(q)z})J_{m\pm2}(qr)J_m(qr')f(r').$$

In paraxial approximation $\beta_1(0) = \beta_2(0) = k\sqrt{\varepsilon_1\mu_1 - \chi^2}$, $\widetilde{\beta}_1 = 2\beta_1(0)\mu_2/\mu_1$, $\widetilde{\beta}_2 = 2\beta_1(0)\varepsilon_2/\varepsilon_1$ should be used.

All solutions obtained above determine the polarization of electromagnetic beams by means of circular vectors $\boldsymbol{e}_\pm$. These vectors form basis in the beam cross-section and allow us to describe any wave polarization (linear, circular, elliptical). Exact formulae for electric field are simplest for isotropic media and very complex for bi-isotropic media. The complexity of the relationships is connected with the nonreciprocal properties of the media: the forward and

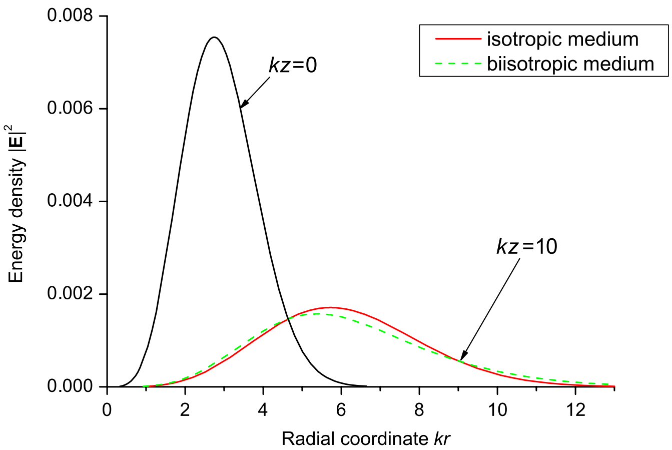

Figure 1. The energy density of the paraxial Bessel–Gauss beam propagating in an isotropic (solid line) and a bi-isotropic (dashed line) media. Isotropic medium is characterized by the values $\varepsilon = 2.1$, $\mu = 1$. Bi-isotropic medium takes the following values: $\varepsilon = 2.1$, $\mu = 1$, $\chi = 0.3$. Beam parameters: $A = 1$, $q_0/k = 0.5$, $kv_0 = 3$, $m = 2$.

backward beams propagate in different ways. The very complicated paraxial solution is the solution for a beam in a bianisotropic medium, because it contains the cross terms with products of the Bessel functions of the orders $\nu - 1$ and $\nu + 1$. The incident cylindrically symmetric beam maintains its cylindrical symmetry only in isotropic media. Polarization of the beam in isotropic media does not change during the beam propagation, too. In bianisotropic media the beam can retain its symmetry in paraxial approximation. The example of such electromagnetic beam is paraxial solution (50) in bi-isotropic media, because the beam energy density does not depend on the azimuthal coordinate: $|\boldsymbol{E}_\perp(r, \varphi, z)| = \left(\left|g_m^{(1)}\right| + \left|g_m^{(2)}\right|\right)/2$.

5. Vector Bessel–Gauss beams

Let us investigate some characteristics (intensities and polarizations) of cylindrically symmetric beams (35) by the example of Bessel–Gauss beam with the following field distribution in cross-section $z = 0$:

$$f(r) = AJ_m(q_0 r)\exp\left(-\frac{r^2}{v_0^2}\right), \tag{58}$$

where A and q_0 are constants, v_0 is the beam waist. In paraxial approximation the function g_m equals

$$g_m(r, z) = \frac{Av_0^2}{Q(z)}\exp\left(\mathrm{i}knz - \frac{\mathrm{i}q_0^2 v_0^2 z}{2knQ(z)} - \frac{r^2}{Q(z)}\right) J_m\left(\frac{q_0 v_0^2 r}{Q(z)}\right), \tag{59}$$

where $Q(z) = v_0^2 + 2\mathrm{i}z/(kn)$. Number n can take the following values: in isotropic media $n = \sqrt{\varepsilon\mu}$ is the refractive index, in bi-isotropic media $n_1 = \sqrt{\varepsilon\mu} + \chi$ for the function $g_m^{(1)}$ and $n_2 = \sqrt{\varepsilon\mu} - \chi$ for the function $g_m^{(2)}$, in bianisotropic media $n_1 = \sqrt{\varepsilon_1\mu_1 - \chi^2\mu_2/\mu_1}$ for the function $g_m^{(1)}$ and $n_2 = \sqrt{\varepsilon_1\mu_1 - \chi^2\varepsilon_2/\varepsilon_1}$ for the function$g_m^{(2)}$. The functions $g_m^{\pm}$ in

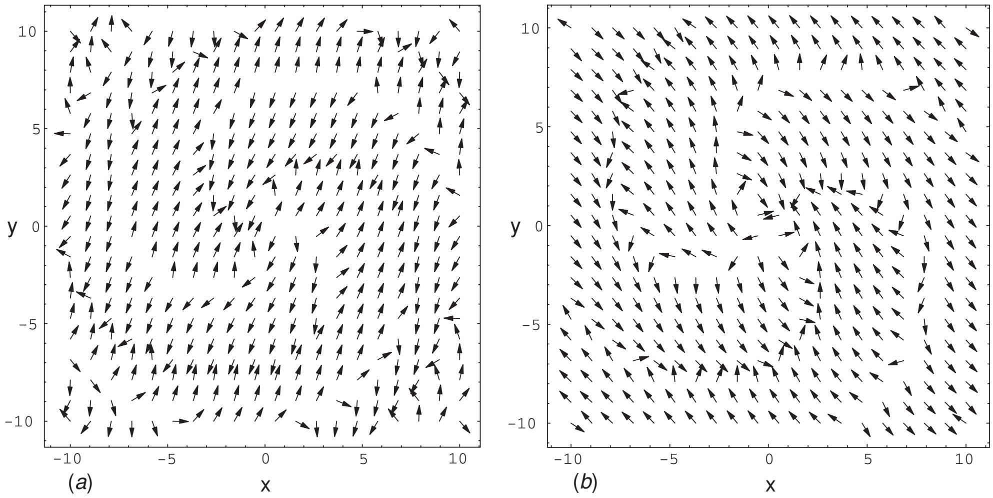

Figure 2. The real part of the electric field strength of the Bessel–Gauss beam in a bi-isotropic medium at distances (*a*) $kz = 10$ and (*b*) $kz = 20$. The length of the arrows is fixed, that is why they show only the direction of the field strength, but not its magnitude. Parameters: $\varepsilon = 2.1$, $\mu = 1$, $\chi = 0.1$, $A = 1$, $q_0/k = 0.5$, $kv_0 = 3$, $m = 2$.

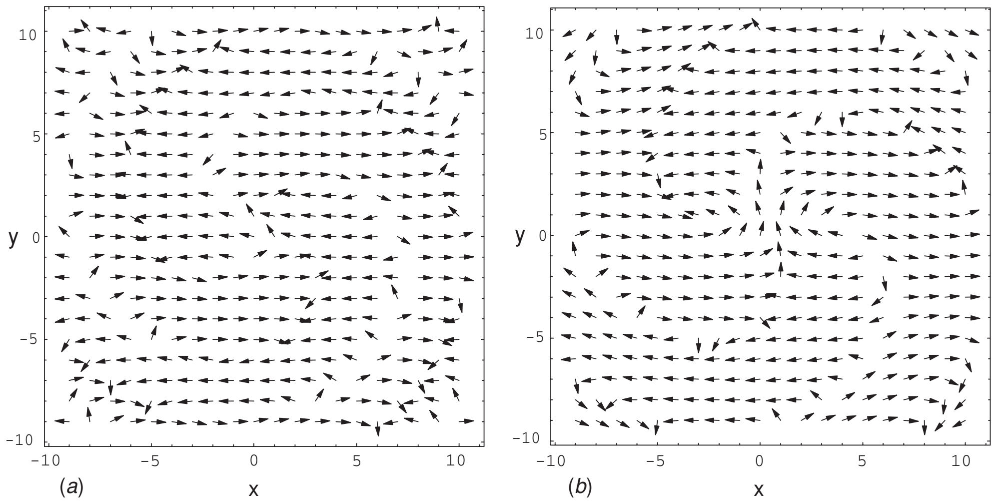

Figure 3. The real part of the electric field strength of the Bessel–Gauss beam in a bianisotropic medium (52) at distances (*a*) $kz = 10$ and (*b*) $z = 20$. Arrows show only the direction of the field strength, but not its magnitude. Parameters: $\varepsilon_1 = 2.1$, $\varepsilon_2 = 2.25$, $\mu_1 = 1$, $\mu_2 = 1$, $\chi = 0.1$, $A = 1$, $q_0/k = 0.5$, $kv_0 = 3$, $m = 2$.

bianisotropic media are of the form

$$g_m^{\pm}(r, z) = \frac{Av_0^2}{2}\,\mathrm{e}^{\mathrm{i}kz\sqrt{\varepsilon_1\mu_1-\chi^2}-q_0^2v_0^2/4}\int_0^{\infty} q\,\mathrm{d}q\left(\mathrm{e}^{-\mathrm{i}q^2z/(2n_1)} - \mathrm{e}^{-\mathrm{i}q^2z/(2n_2)}\right) \times \mathrm{e}^{-q^2v_0^2/4}J_{m\pm2}(qr)I_m\left(\frac{qq_0v_0^2}{2}\right). \tag{60}$$

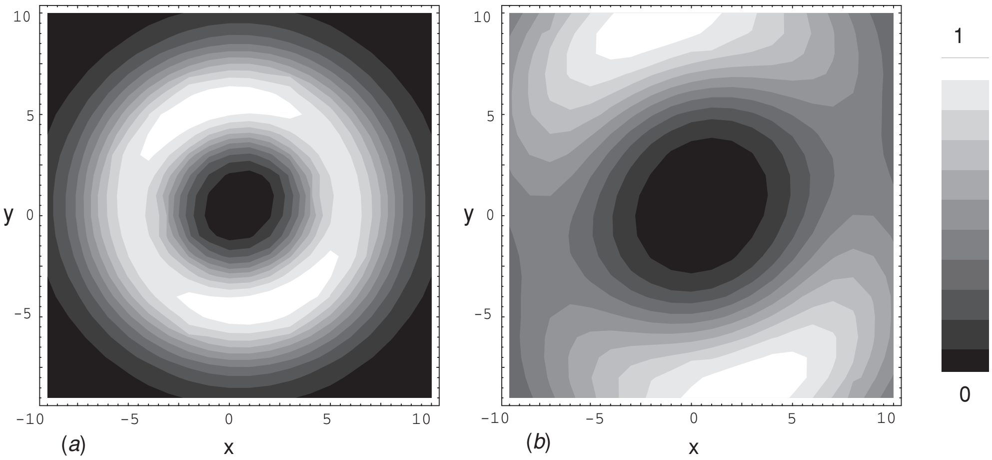

Figure 4. Normalized energy density of the Bessel–Gauss beam in a bianisotropic medium (52) for distances (*a*) $z = 10$ and (*b*) $kz = 20$. Beam and media parameters are the same as in figure 3.

Paraxial cylindrically symmetric beams maintain their cylindrical symmetry in isotropic and bi-isotropic media (see previous section). This property is demonstrated in figure 1. The incident Bessel–Gauss beam strongly diffracts moving in z direction. The difference in the intensity profiles is small for the beams in isotropic and bi-isotropic media. The main difference for the beams is connected not with the field magnitude, but with polarization (vector field direction). In isotropic media linearly polarized incident beam maintains its polarization, while in bi-isotropic media the polarization varies in the beam cross-section and significantly changes during propagation. In figure 2 there are the regions in the beam cross-section, in which the beam is predominantly right or left circular polarized.

In the bianisotropic medium linearly polarized Bessel–Gauss beam mainly retains its linear polarization due to the influence of the magnitudes $g_m^{(1)}$ and $g_m^{(2)}$ in expression (56). The rest summands in this formula are essential only when the sum $g_m^{(1)} + g_m^{(2)}$ tends to zero (it is, for example, the beam centre in figure 3). The energy density of the Bessel–Gauss beam is not cylindrically symmetric. During beam propagation the field maxima shift to the periphery and become more indicative, while the field distribution gets the form of ellipse (see figure 4).

6. Conclusion

The offered approach for beam constructing can be applied to the wide class of media described by expression (2). An important place in the approach takes the tensor notation of the transverse field strengths of cylindrical eigenwaves (16), because it allows us to separate the initial field vector from the coordinate dependence. To construct the beam with arbitrary field distribution one should use the superposition of partial waves, i.e. the Fourier transform. There is the tensor link between the electric field and its Fourier transform. Such link is called tensor Fourier transform. The tensor Fourier transform arises naturally, because the tensor characteristics of the media enter the Maxwell equations.

An arbitrary beam can be constructed as the superposition of a plane or spherical partial waves, too. That is why in spite of the fact that the obtained exact formulae of the spatial field evolution (30) are applicable to arbitrary beams, one awaits the wide use of these expressions in optics of cylindrically symmetric and azimuthally modulated beams.

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